5. Modelling a basketball throw

teacher guide

Goal of the task

The main aim is to bring a familiar "real-life" application into the mathematics classroom an provide the need for the use of linear and quadratic functions. The task also helps to relate skills of data-handling (statistics) to those of algebra.

Target group and required time

All these prerequisites are generally (more or less) well known by students of 11th grade after the first semester or 10th grade at the end of the academic year. The task requires approximately 3 to 5 lessons in the computerlab, depending on whether the additional task is completed.

Preliminary TI InterActive! skills

The students need some basic skills in using TI InterActive!, in particular the use of the Data Editor, Statistical plots (scattergraphs), formatting graphs (axes and labels), superimposing functions on graphs and performing statistical calculations such as linear and quadratic regressions. Students familiar with the underlying theory of motion under gravity may use the Math Box to solve equations and/or compute derivatives.

Preliminary mathematical skills

Students should be familiar with the notion of a displacement-time graph.

Students need to appreciate how the shape of a scattergraph can suggest the form of an equation of a curve of good fit. Students need to understand the effect of the parameters a ,b in the linear function $x = a + bt$. They should also have met transformations of the quadratic function in the form: $y = c + d.(t - e)^2$ and be able to relate these to translations and change of scale.

Students should have met the basic idea that statistical software can compute "best-fit" functions to represent data but they need not have knowledge of the algorithms used (or what "least-squares" means).

Students who have studied some physics or applied mathematics should be able to relate the linear and quadratic functions to the equations of motion under constant acceleration and may be in a position to use simple methods of differential calculus.

File organization

The task consists of the following linked TII files:

- Notebook.tii: The file where the student writes the solutions and the starting point containing hyperlinks to the other files.
- MainTask.tii: Describes the task and contains hyperlinks to Hint files and Help files.
- HelpA1.tii and HelpA3.tii: contain some technical help to perform some of the required tasks of the Main Task.
- HintA2.tii and HintB2.tii: Contain mathematical suggestions on fitting functions "by eye".
- AdditionalTask.tii: Contains an additional task for the brighter or faster student.
- Solution.tii: file Contains possible solutions for each of the tasks.
- GeomScreen.tii: A file containing two screen dumps to illustrate possible approaches using Dynamic Geometry Systems.

The hyperlinks only work if the files are installed in the map c:\TII\Basketball. Ideally, the teacher should delete the link that exists from the Notebook file to the Solution file before the students start to work!

Classroom organisation

The lesson can be introduced by the teacher using a computer displayed through a data-projector. The follow-up work involves the students working either individually or in pairs at computers during the normal mathematics classes or in their own time. The above files are organized and detailed in such a way that the students should be able to work without the presence of their mathematics teacher.

Didactical suggestions

In order to set the scene it might be useful to arrange to have a basketball in the class to give the idea of a "trajectory". Then using software such as Windows Media Player the teacher can project the video clip "Basket.avi" onto a whiteboard and students can mark positions of the ball in flight.

Technical hints

Students start by opening the notebook file in TII. It is recommended that the main task is opened in a separate window so that two versions of TII are activated. By means of a right mouse click in the Windows menu bar the option 'Windows Cascade' offers opportunities for switching easily between the files. However, if the same hyperlink is used twice two versions of the same file will be opened! This may be a source of confusion and therefore requires some attention by the teacher.

In case the data in the table in the Main Task file are different from the ones on page 63, please consider changing you Windows regional setttings to English (US) and install the TII student units files again.

As TII only supports absolute hyperlinks and no relative links, the links need to be adjusted after installation of the files, for example on the school network.

5.1 Main task

intro

The main task consists of three exercices (A, B, C) in which you will have to use, among others features, the stat plot tool, function plotting and statistical calculations. If you do not feel familiar with one of the above tools then consult the help links provided in this document or use the TI InterActive! Help icon .

The mathematical skills you need to do the exercices are related to distance-time graphs, velocity and acceleration, linear and quadratic functions and their graphs. Some links to mathematical hints are also provided.

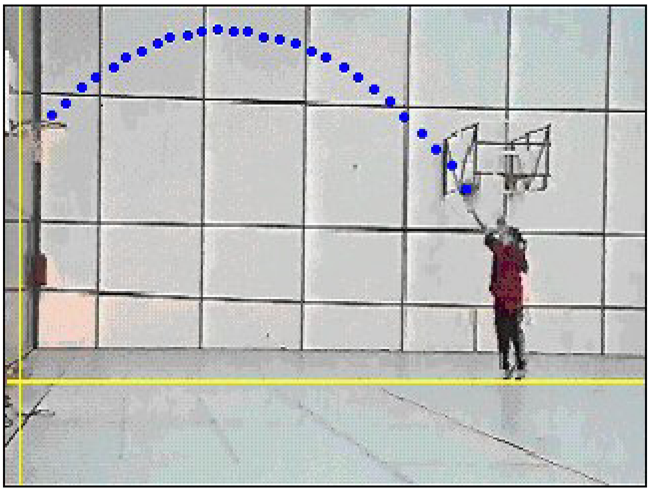
task

Description

The figure below shows some of the positions of a basketball during a shot at the basket.

Using the floor as the *x*-axis and the wall as the *y*-axis the co-ordinates of the ball, together with the elapsed time were captured using the Vidshell 2000 software by Doyle V. Davis which can be downloaded from: http://webphysics.tec.nh.us/vidshell/vidshell.html

The video clip is in the file Basket.avi and comes from the DAVID project at the University of Munich, Germany.



x	y	t
5.15	2.22	.00
4.99	2.50	.04
4.81	2.68	.08
4.64	2.87	.12
4.44	3.05	.16
4.27	3.23	.20
4.09	3.37	.24
3.90	3.51	.28
3.74	3.63	.32
3.54	3.74	.36
3.37	3.81	.40
3.19	3.90	.44
3.00	3.95	.48
2.82	3.97	.52
2.63	4.04	.56

The *x*- and *y*-coordinates are measured in metres from the origin which is below the basket. The time values are measured in seconds (the video has 25 frames per second.)

Task A

a. Draw the scattergram of horizontal displacement *x* against time *t* . Choose suitable scales for the axes and include labels. **Help 1**

b. Fit a linear functionof the form $p(t) = a + b.t$ to the data "by eye". **Hint 1**

c. Find an appropriate regression model for *x* against *t*. **Help 2**

Task B

a. Draw the scattergram of vertical displacement *y* against time *t*. Choose suitable scales for the axes and include labels.

b. Fit a quadratic function of the form $q(t) = c + d\,(t - e)^2$ to the data "by eye". **Hint 2**

c. Find an appropriate regression model for *y* against *t*.

Task C

a. Draw the scattergram of vertical displacement *y* against horizontal displacement *x*. Choose suitable scales for the axes and include labels.

b. Fit a quadratic function of the form $r(x) = f + g\,(x - h)^2$ to the data "by eye".

c. Find an appropriate regression model for *y* against *x*.

5.2 Additional task

task

Velocity and Acceleration

If you ignore air-resistance, spin etc., the only force acting on the ball during its flight is the Earth's gravitational attraction which is in the vertical direction. Now you will see what information you can draw from the linear and quadratic mathematical models you constructed in the Main Task.

Remember that with the co-ordinate axes used in this task positive *x* is measured to the right and positive *y* is measured upwards. Take care to give the correct units for velocities and accelerations and to make sure that they have the correct sign (positive or negative).

a. Horizontal velocity

From the linear regression model for the horizontal motion:
estimate the value of the (constant) horizontal velocity.

b. Vertical velocity and acceleration

From the quadratic regression model for the vertical motion:

(i) find its derivative to give a linear model for the vertical velocity,
(ii) estimate the value of the vertical velocity at time $t = 0$,
(iii) estimate the value of the vertical velocity when the ball enters the basket,
(iv) estimate the value of the (constant) vertical acceleration *g*.

c. Velocity vectors

Using any of the three models and the information derived above, find:

(i) the velocity vector u of the ball at the point of release ($t = 0$),
(ii) the angle *a* that the vector u makes with the horizontal,
(iii) the magnitude of the vector u,
(iv) the velocity vector v of the ball as it enters the basket,
(v) the angle *b* that the vector v makes with the horizontal,
(vi) the magnitude of the vector v.

d. Validation of angles

Using geometric instruments, such as a ruler and protractor,
check your answers to c(ii) and c(v) against the picture on page 1 of the Main Task.

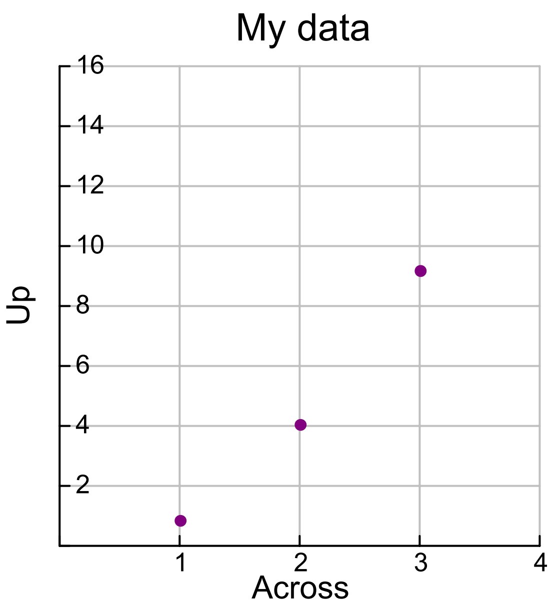
5.3 Hints

hint 1

Fitting a linear function to data "by eye"

Suppose you have some data in lists and have plotted a scattergraph from them.

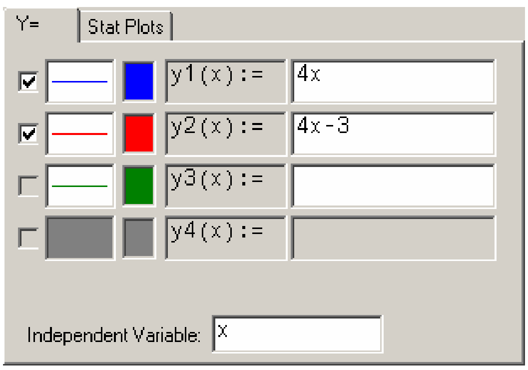
L1	L2
1	0.9
2	4.1
3	9.2

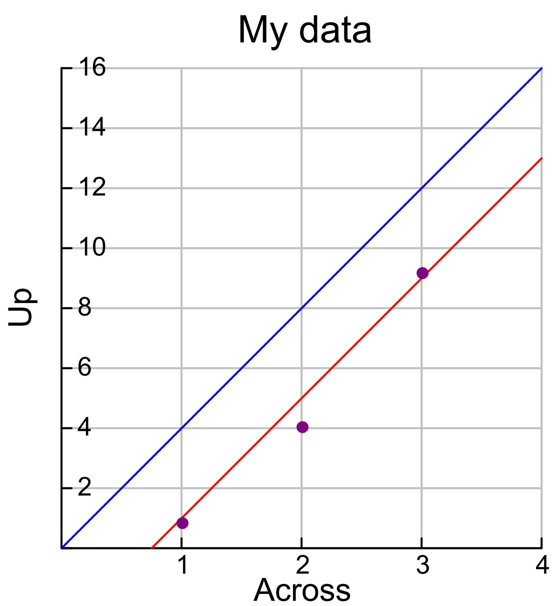


You can use the f(x) Tab from the Functions dialog box to enter the equation of a function whose graph you want to to display over the scattergraph.

For example my first estimate of a suitable linear function might be $y = 4x$ which is entered in the first box, and plotted as a blue line.

I might think that a better model would be a line which was closer to the points, and I can enter another choice e.g. $y = 4x$-3 in the second box and plot it as a red line. In this way you can build up better estimates of functions which give a "good fit" to the data.

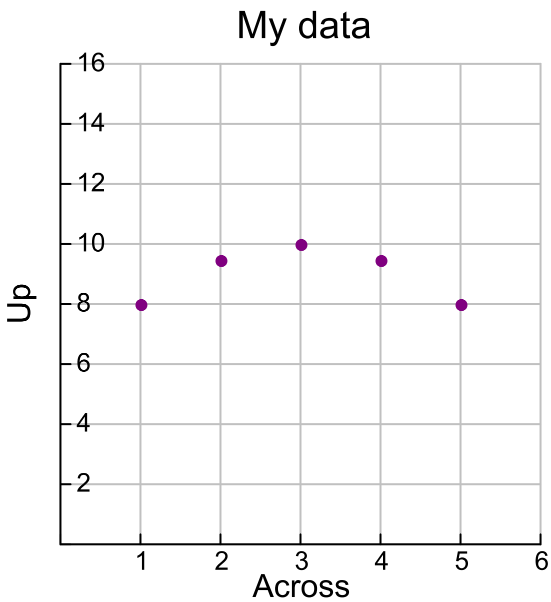




Fitting a quadratic function to data "by eye"

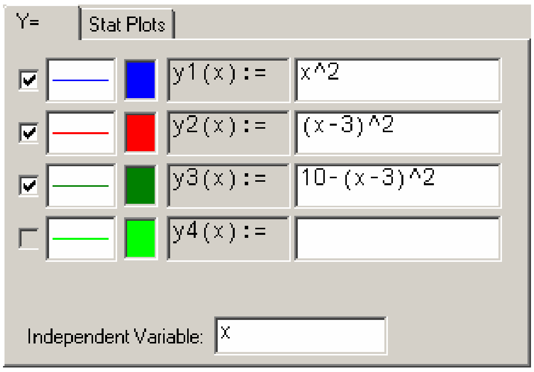
Suppose you have some data in lists and have plotted a scattergraph from them.

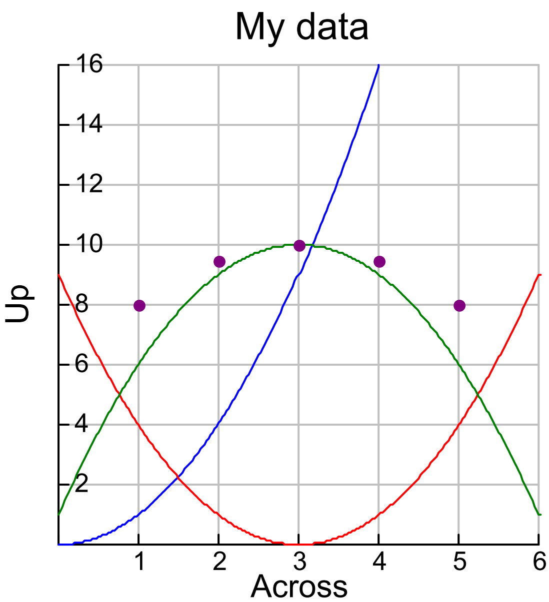
L1	L2
1	8
2	9.5
3	10
4	9.5



You can use the f(x) Tab from the Functions dialog box to enter the equation of a function whose graph you want to to display over the scattergraph.

In this case it looks as if a quadratic function might give a good fit. The vertex is at (3,10) and the curve slopes donwards so we need to find transformations of the basic quadratic function $y = x^2$ to givea curve with the desired characteristics.





The blue graph is for $y = x^2$.

The red graph is for $y = (x - 3)^2$.

The green graph is for $y = 10 - (x - 3)^2$.

Can you find the equation of a quadratic function which passes through all 5 data points?

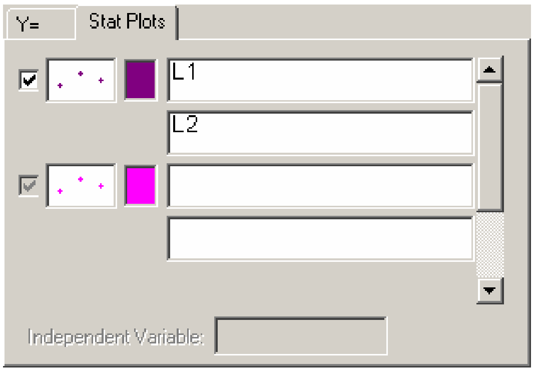
5.3 TI InterActive! help

Drawing a scattergraph from data in lists

L1	L2
1	0.9
2	4.1
3	9.2

With some data entered in lists, such as L1 and L2, highlight the list you want plotted on the *x*-axis by clicking on the name e.g. L1 at the top. To select the list for the *y*-axis, hold down the CTRL key while clicking the name, for example L2, at the top.

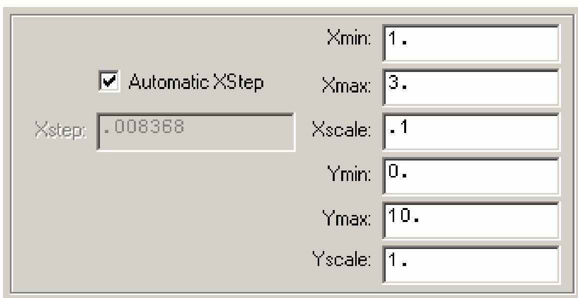
Then double click  on the Graph icon in the toolbar.

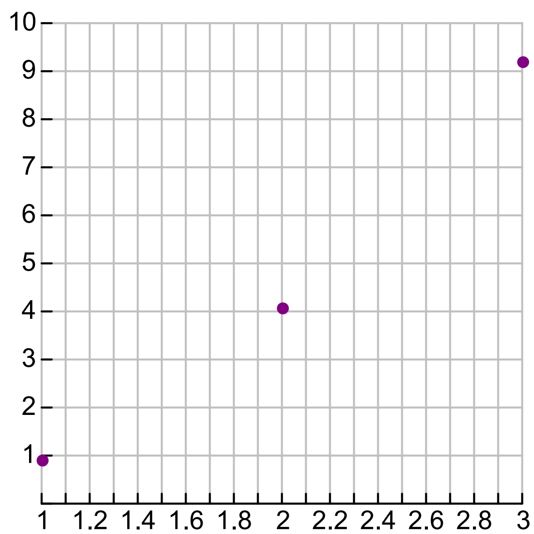


This brings up the Functions window which shows that the lists L1 and L2 have been selected and that a scattergraph will be drawn. The scales for the axes have been chosen by the software.

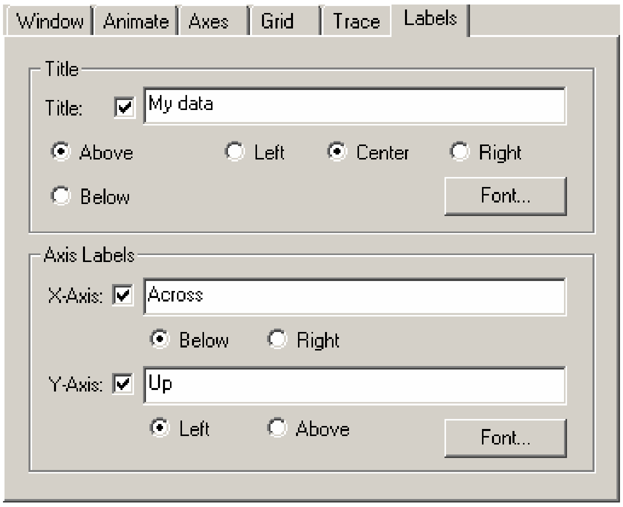
Click on the brown FORMAT button to bring up the dialog box for you to enter your own preferences.

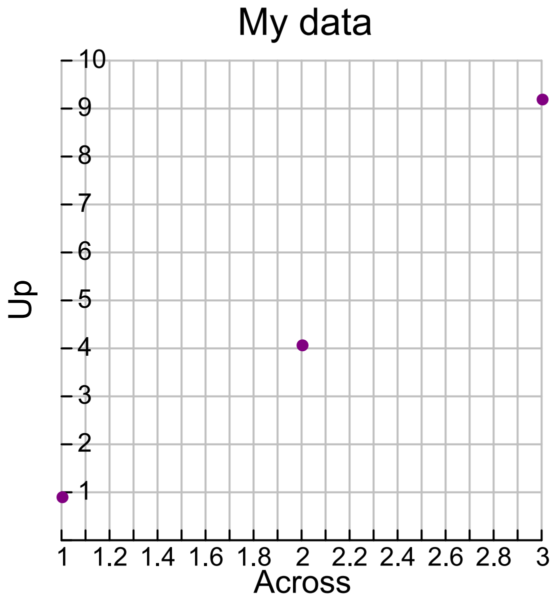
From the Window tab you can enter your own choices for the Minimum, Maximum and Step values for each of the axes.





Clicking on the Labels tab you can give the graph a title and label each of the axes.





help 2

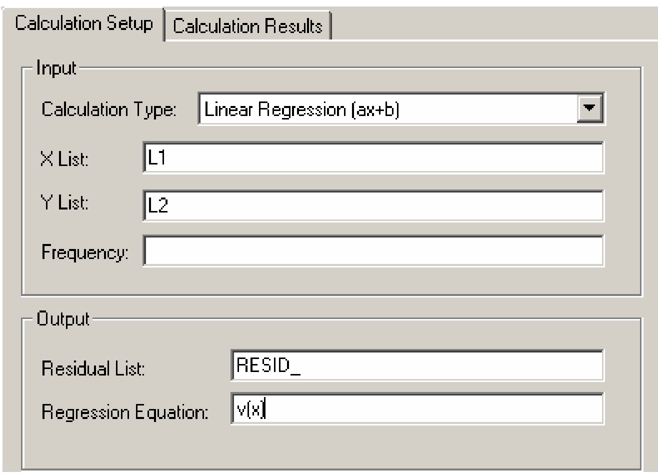
Fitting a regression model to data in lists

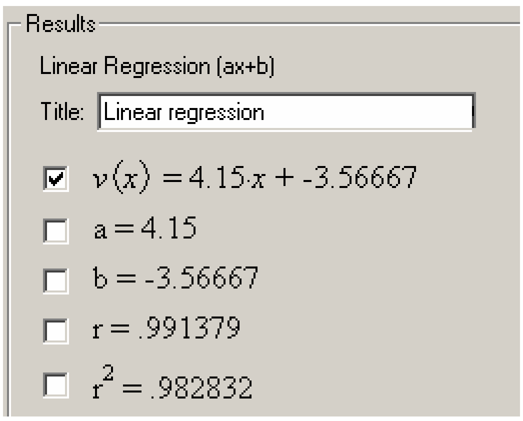
L1	L2
1	0.9
2	4.1
3	9.2

With some data entered in lists, like in L1 and L2, highlight the list you want for the independent variable by clicking on the name e.g. L1 at the top. To select the list for the dependent variable hold down the CTRL key while clicking in the name e.g. L2 at the top.

Then double click on the StatCalc icon, , in the toolbar.

Click on the arrow alongside the "Calculation Type" box and select from the list, which in this case is Linear Regression. The names of your selected lists should appear in the X-list and Y-list boxes, but you can type them in or edit them if you prefer. In the last box type a sensible name to be used for the regression function. Then click the Calculate button to see what the software computes as the regression ("best fit") function. You can choose which parts of the computation will be saved to your document.





Now you can open the Grapher and enter the name of the calculated regression function e.g. v(*x*) to plot its graph. Here the two "by eye" functions from previous work are shown as dotted lines and the regression function by the blue graph.